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POTENTIAL APPLICABILITY OF MESHFREE METHOD USING LAGRANGE MULTIPLIER

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ABSTRACT

An example has been illustrated to demonstrate the potential application of EFG mesh – free method without the use of background mesh. MLS procedure is deployed to arrive at the shape function. Lagrange Multiplier method is used to enforce the boundary condition. An algorithm based on MATLAB coding is developed to obtain displacement profile along the length of the 2-D cantilever. The result obtained shows good agreement with the analytical solution.

KEYWORDS: Mesh-Free Method, Moving Least Square Method, Lagrange Multiplier

INTRODUCTION

All the physical phenomena encountered in engineering are modelled by differential equations. To solve the differential equations, two major approaches are followed - Analytical and Numerical. Analytical approach leads to closed-form solutions and is effective in case of simple geometry, boundary conditions, loadings and material properties. For most of practical problems where it is not possible to get exact analytical solution, numerical methods are being called for. The various numerical methods available are: FDM, BEM, FVM, FEM, X-FEM, and Mesh Free Method. Strong form method discretizes and solves the governing differential equation directly, solution being more accurate at nodal points. Weak form method instead of solving differential equation of the underlying problem directly; an integral function that governs the same physical phenomena is solved, resulting solution in an averaged sensed. This paper gives a detailed analysis of a 2D cantilever with a point load at free end using Mesh Free analysis.

NECESSITY OF MESHFREE METHOD

Though FEM is a robust and thoroughly developed method, and widely used in engineering fields due to its versatility for complex geometry and flexibility for many types of linear and non-linear problems and also most practical engineering problems are currently solved using well developed FEM packages, it has lots of drawbacks.

High cost in creating an FEM mesh: An analyst needs to spend most of his time in creating a quality mesh as the computer cannot always create a quality mesh.

Remeshing in FEM requires a complex, robust and adaptive mesh generation processors, which are workable only for 2D problems.

Low accuracy of stresses: Since the displacement functions are piecewise continuous, the stresses obtained in FEM packages will be discontinuous at the interface and won't be accurate.

FEM gives lower bound solution to the exact solution whereas mesh free can produce lower as well as upper bound solutions.

FDM works well only for regularly distributed nodes. Studies are still going on to develop methods using irregular grids.

The root of these problems is the use of elements or mesh in the formulation stage. The idea of getting rid of the elements and meshes in the process of numerical treatments has naturally evolved, and the concepts of mesh free or mesh less methods have been shaped up. In MeshFree, there is no need to create a quality mesh and the nodes can be created by a computer in a much more automated manner, much of the time an engineer would spend on conventional mesh generation can be saved. This can translate to substantial cost and time savings in modeling and simulation projects.

MESHFREE METHOD

It is a method used to establish a system of algebraic equations for the whole problem domain without the use of a predefined mesh or uses easily generable meshes in a much more flexible or freer manner. It uses a set of field nodes scattered within the problem domain as well on the boundaries of domain to represent the problem and its boundaries.

The brief procedure is explained in the flow chart

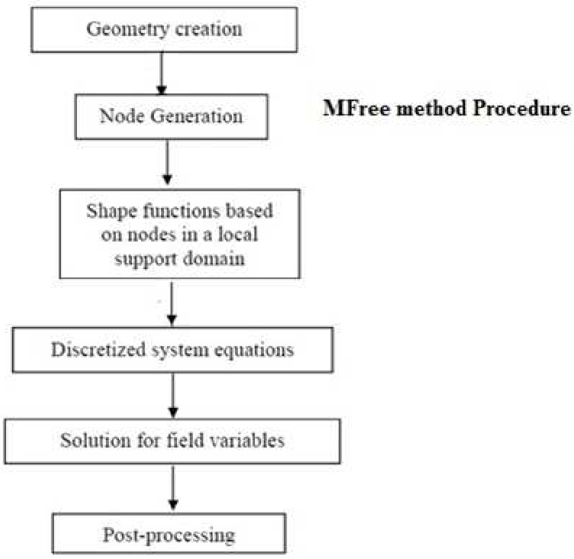

Figure 1: Flow Chart of the MFree Method's Procedure

In this work shape functions are constructed using Moving Least Square (MLS) method.

MOVING LEAST SQUARES SHAPE FUNCTIONS

The moving least squares (MLS) approximation was devised by mathematicians in data fitting and surface construction (Lancaster and Salkausdas 1981; Cleveland 1993). It can be categorized as a method of series representation of functions. The MLS approximation is now widely used in Mesh Free methods for constructing Mesh Free shape functions.

Formulation

Shape function or interpolation of field variables decides the accuracy of the results. u(x, y) is the function of field variable defined in the domain. If the approximation of u(x, y) at a point is given as uh(x, y), then the MLS approximation can written as,

Uh(x) = $\sum_{j}^{m} p_j(x) a_j(x)$ = $p^T(x)$ a(x)

Where p(x,y) is the basis function of the spatial coordinates, and m is the number of the basis functions. The basis function p(x,y) is often built using monomials from the Pascal triangle to ensure minimum completeness.

a(x) is the vector of coefficients given by

$a^T(x)$ = { $a_0(x)$ $a_1(x)$ ….. $a_m(x)$} which are functions of x. The coefficients a can be obtained by minimizing the following weighted residual function.

J = $\sum_{I}^{n} W\ (x - x_I)[u^h(x, x_I) - u(x_I)]^2$

= $\sum_{I}^{n} W\ (x - x_I)[p^T(x_I)a(x) - u_I]^2$

Where $W\ (x - x_I)$ is a weight function, chosen so that to have the following properties

$W\ (x - x_I)$> 0 within the support domain

$W\ (x - x_I)$ = 0 outside the support domain

$W\ (x - x_I)$Monotonically decreases from point of interest x

$W\ (x - x_I)$is sufficient, smooth , especially on the boundary

Exponential weight function is chosen here.

$W_I(x) = \begin{cases} e^{(r_i/\alpha)^2} & r_i \leq 1 \\ 0 & r_i > 1 \end{cases}$

Where α=0.3 and $r_i = d_i / r_w$

d_i is the distance between point of interest and the node considered.

r_w is the size of the support domain.

n is the number of nodes in the support domain of x for which theweight function $W\ (x - x_I) \neq 0$ and u_i is the nodal parameter of u at x=x_iEquation (3.125) is a functional, a weighted residual, that is constructedusing the approximated values and the nodal parameters of the unknownfield function.

The stationary of J with respect to a(x) gives

$$\frac{\partial J}{\partial a} = 0$$

which leads to,

$\Phi^T(x) = \{\Phi 1(x), \Phi_2(x), \ldots, \Phi_n(x)\}_{(1 * n)} = p^T(x) * A^{-1}(x) * B(x)$

Where $\Phi(x)$ is shape function

$A(x) = \sum_{i=1}^{n} W_i(x)p(x_i)p^T(x_i)$

$B(x) = \sum_{i=1}^{n} W_i(x)p(x_i)$

$p(x_i) = [1; x_i; y_i; x_i y_i]$

Using Lagrangian multiplier technique the final equation can be obtained as

$KU + G\lambda - F = 0$

$G^T\lambda - q = 0$, by solving which we can get the final displacements of the problem considered.

In matrix form,

$$\begin{bmatrix} K & G \\ G^T & 0 \end{bmatrix}\begin{pmatrix} U \\ \lambda \end{pmatrix} = \begin{pmatrix} F \\ q \end{pmatrix}$$

$K_{IJ} = \int B_I^T D\, B_J$

$$B_I = \begin{pmatrix} \varphi_{I,x} & 0 \\ 0 & \varphi_{I,y} \\ \varphi_{I,y} & \varphi_{I,x} \end{pmatrix}$$

$G_{IJ} = -\int N_I^T \varphi_J^T d\Gamma$

$$\varphi_I^H = \begin{bmatrix} \varphi_I^H & 0 \\ 0 & \varphi_I^H \end{bmatrix}$$

$$N_I = \begin{bmatrix} N_I & 0 \\ 0 & N_I \end{bmatrix}$$

K_{IJ} – nodal stiffness matrix which is assembled to get the global stiffness matrix (K)

φ_I^H – Shape function matrix (for 2 DOF considered).

λ - Lagrange multiplier.

N_I - Lagrange interpolant used in the conventional finite element method (FEM).

Γ – Element's essential boundary.

PROBLEM DEFINITION

A 2D cantilever ABCD of dimensions 2500mm * 500mm is considered. AB is fixed and a load of 40 kN is applied on free end. The material properties considered are, Young's modulus E = 25000 N/mm^2, Poisson's' ratio = 0.15.

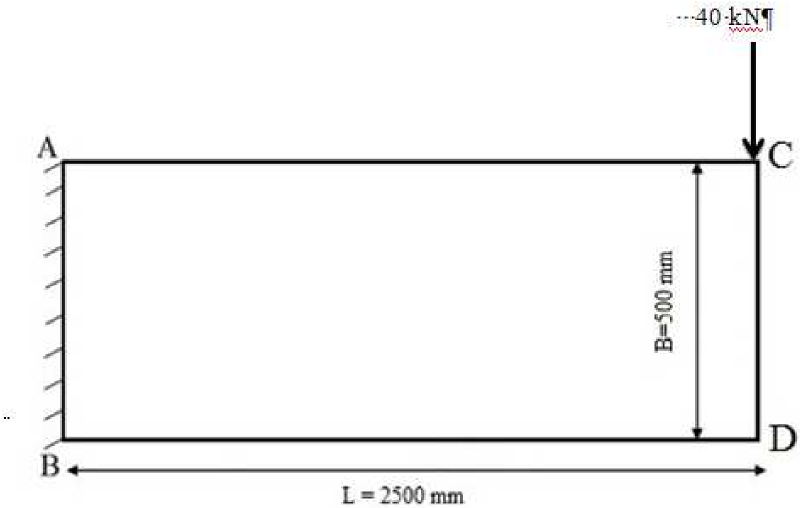

Figure 2: D Cantilever with Point Load at Free End

RESULTS& DISCUSSIONS

Results of Parametric Study

A Parametric study was conducted on different nodal combination by varying the spacing of nodes.The nodes are distributed in triangulated fashion in all the following exercises. The variation of the displacement along the length of the cantilever is captured and it is compared with analytical solution.

Exercise 1: Nodal spacing = 250 mm (Total number of nodes = 32)

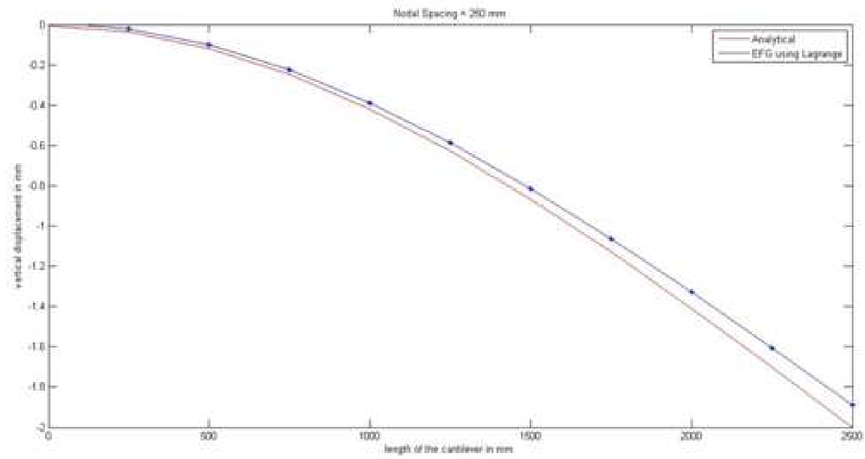

Figure 3: Comparison of Mfree Solution with Analytical Solution

Exercise 2: Nodal spacing = 125 mm (Total number of nodes = 103)

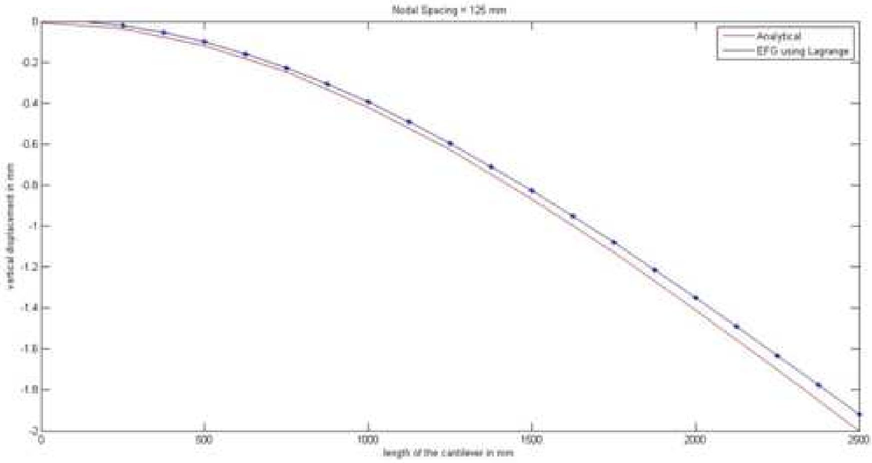

Figure 4: Comparison of Mfree Solution with Analytical Solution

Exercise 3: Nodal spacing = 100 mm (Total number of nodes = 155)

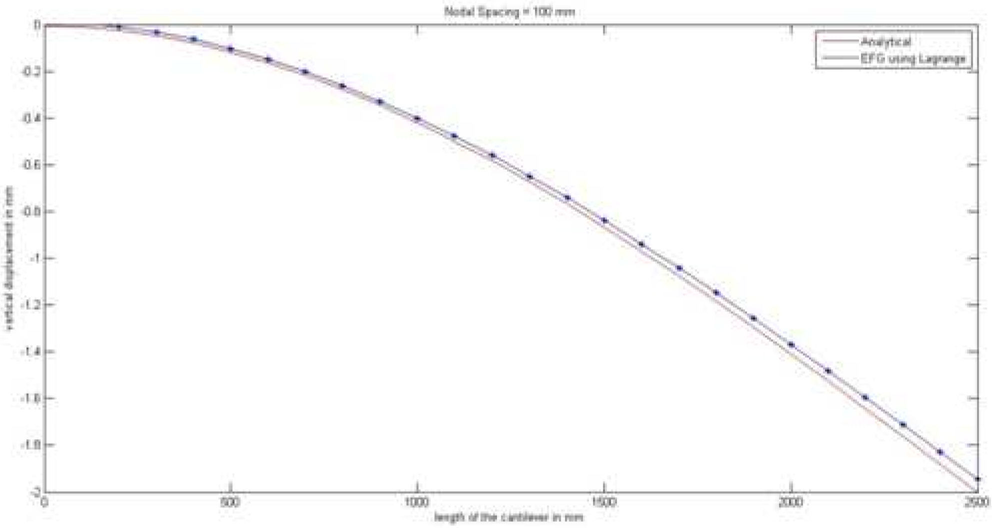

Figure 5: Comparison of Mfree Solution with Analytical Solution

Exercise 4: Nodal spacing = 62.5 mm (Total number of nodes = 365)

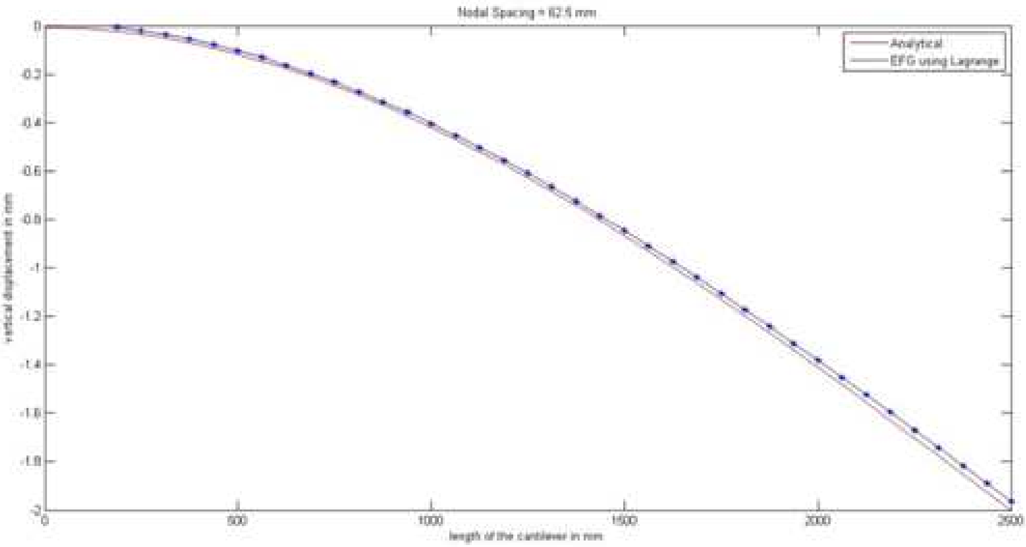

Figure 6: Comparison of Mfree Solution with Analytical Solution

Exercise 5: Nodal spacing = 50 mm (Total number of nodes = 556)

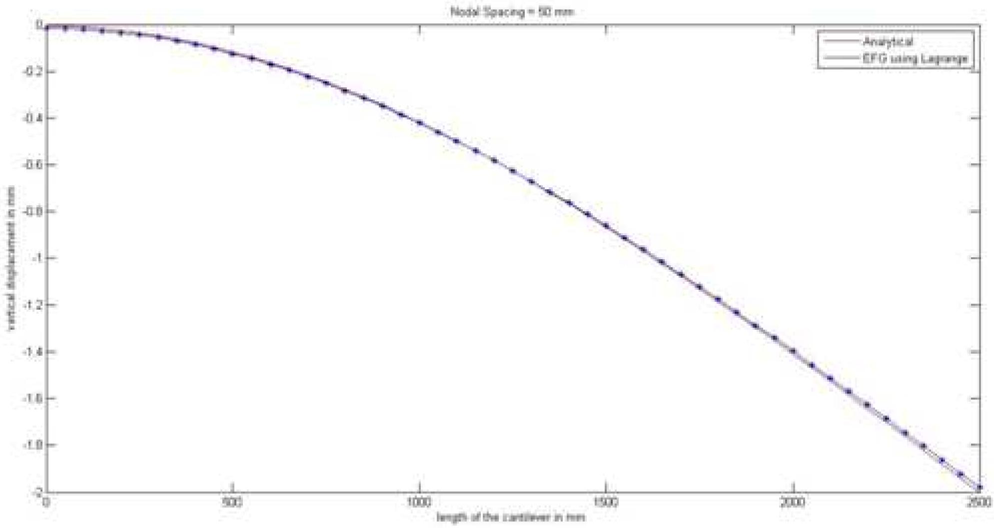

Figure 7: Comparison of Mfree Solution with Analytical Solution

DISCUSSIONS

It is clear from that above graphs, as the number of nodes increases (spacing between the nodes decreaces), the accuracy of the result increases.

The displacement at the free end for all the node combination is taken and compared with analytical solution in the following graph. As the number of nodes increases the MFree solution moves closer to the analytical solution. In our problem, while taking for 50 mm nodal spacing (i.e 556 nodes) Mfree solution matches almost with the analytical solution. Therfore it is concluded that further increase of nodes will result only in increasing the computational time but not in improving the accuracy too much.

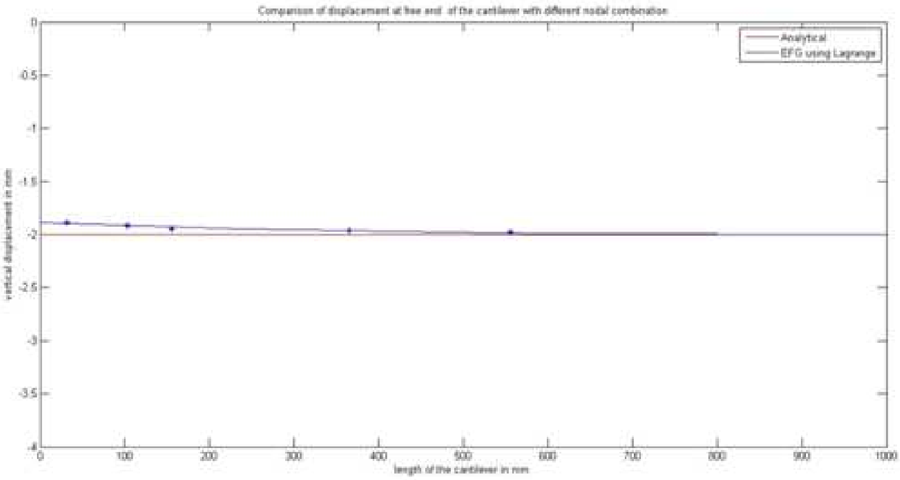

Figure 8: Comparison of Mfree Solution with Analytical Solution for the Displacement at Free End

CONCLUSIONS

- This procedure doesn't use any background mesh (even for nodal integration) hence it is a purely meshless method.
- The complexity accompanied with the use of Gaussian quadrature is eliminated.
- With the increase in number of nodes, Mesh free solution approaches analytical solution.
- The degree of refinement is fixed by conducting a parametric study for different nodal combinations using the output obtained.

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